Wide-Range Electrostatic Loudspeaker with a Zero-Free Polar Response*

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The design of the floor-to-ceiling electrostatic doublet, which was partially developed by Baxandall, is completed by using a symmetrical *RC* transmission line to equalize the frequency response. The combination of tall narrow stator segments and the *RC* transmission line gives the loudspeaker a broad zero-free polar response that is close to the ideal cosine response of a dipole radiator at all audio frequencies. The equalization also ensures that the whole ESL panel contributes to the acoustic output at low frequencies. Reflections from the floor and ceiling are an integral part of the floor-to-ceiling design.

0 INTRODUCTION

Since the late 1950s, when the Acoustical Manufacturing Company (now Quad, UK) marketed their wide-range electrostatic loudspeaker (ESL), ESLs have been held in high regard. With due attention to basic design principles, ESLs are capable of midband distortion levels well below those of conventional electromagnetic loudspeakers [1]. However, ESLs also have a reputation for a limited bass response, a poor polar response, and for presenting a difficult low-impedance capacitive load to power amplifiers.

One little exploited advantage of ESLs is that the sound pressure level (SPL) is calculable from the stator current [1], [2]. This relationship, the Walker equation, led to the development of the Quad ESL-63 loudspeaker. In the ESL-63 the stator is composed of concentric annular segments, each forming the capacitive section of an *LC* transmission line [2], [3]. The transmission line presents a nominally resistive impedance to the amplifier, and introduces delays between the responses of the stator segments to mimic a point source, giving the ESL a well-defined polar response largely independent of the loudspeaker dimensions. Unfortunately commercial imperatives limited the number of annular segments used in the design [3]. At frequencies above 3 kHz and off-axis angles exceeding 15°, the small number of segments and their finite widths cause the polar response to deteriorate with narrow dips of 15 dB or more at some angles, indicating the likely presence of zeros in the response [4].

Baxandall, in his review of ESL design [1], partially developed a design he called the floor-to-ceiling doublet. By exploiting reflections from the floor and ceiling, the design mimics an infinitely long line source. The Walker equation for this design, however, has an awkward $f^{1/2}$ frequency dependence, suggesting that it might not be equalized exactly with passive components. Lipshitz and Vanderkooy [5] also noted the likely difficultly of equalizing bipolar line sources.

This engineering report extends Baxandall's doublet design by using an *RC* transmission line for equalization. The *RC* transmission line has the requisite $f^{1/2}$ frequency dependence and, in its symmetric form, gives the ESL an almost ideal zero-free horizontal polar response. The design evolved in part from the transmission-line idea used in the Quad ESL-63, and in part from the *RC* network equalization used by Malme [6] and common in many commercial ESLs. Indeed, Malme noted the improved polar response with the use of the *RC* network and commented on the similarity to an *RC* transmission line. However, in Malme's and most commercial ESLs the ESL does not operate as a line source, so the *RC* network cannot provide accurate equalization for all listening distances. With the floor-to-ceiling doublet the *RC* transmission-line equalization scheme described here provides accurate equalization over a wide range of frequencies, listening distances, and angles.

The first two sections develop the design equations for the frequency response and the polar response of the ESL. Section 3 then briefly addresses additional design considerations relating to the implementation of the ESL. Throughout the text it is assumed that the reader is familiar with the basic principles of ESLs and their construction [1], [3], [7], [8].

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1 FREQUENCY RESPONSE

Fig. 1 shows a simple schematic diagram of an ESL with a thin flexible diaphragm stretched between two perforated-metal stators connected in a push–pull arrangement. The diaphragm has a very high surface resistivity to ensure constant charge operation. If the stators and diaphragm are acoustically transparent, the on-axis SPL for a tall narrow ESL is [1, p. 167]

$$P = \frac{I_{\text{sig}} V_{\text{pol}}}{2\pi d h \sqrt{crf}} \tag{1}$$

where I_{sig} is the stator current, V_{pol} is the polarizing voltage applied to the diaphragm, d is the diaphragm–stator spacing, h is the height of the strip, r is the distance between the strip and the listener, f is the frequency, and c is the speed of sound.

Eq. (1) shows that to obtain a flat frequency response, the stator current must be proportional to the square root of the frequency. At first sight it might seem that the frequency equalization can only be approximate because capacitors and inductors have impedances that depend simply on frequency or inverse frequency. However, an

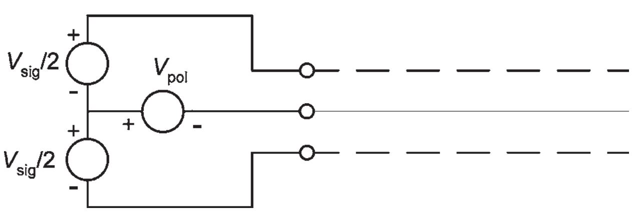

Fig. 1. Simple schematic of ESL.

RC transmission line has a characteristic impedance with the requisite frequency dependence [9],

$$Z_c = \sqrt{\frac{R}{j2\pi fC}} \tag{2}$$

where R and C are the resistance and capacitance, respectively, per unit length of the line, and $j = \sqrt{-1}$ represents a 90° phase shift. Fig. 2 shows a simple schematic diagram of an ESL based on the *RC* transmission line. Each capacitor corresponds to a tall narrow stator segment, of which there are a finite number, and R is the intersegment coupling resistance. If the current through the termination is ignored for the moment (the termination will be discussed in Section 3.1), then all the current flowing into the transmission line eventually flows through one of the stator segments. That is, the total current depends on the signal voltage V_{sig} and the characteristic impedance, and the SPL is

$$P = \frac{V_{\text{sig}} V_{\text{pol}}}{dh\sqrt{2\pi cr}} \sqrt{\frac{C}{R}} \tag{3}$$

which is independent of frequency.

The effect of the series of *RC* filters comprising the transmission line is to gradually attenuate the voltage across the stator segments, with the attenuation increasing with increasing frequency, as shown in Fig. 3. The decrease in voltage is more than offset by the decrease in the impedance of each stator segment (a capacitor) so that there is a net $f^{1/2}$ increase in the stator current, which gives the ESL the flat frequency response.

In practice, because the ESL is composed of a finite number of discrete sections, the ESL has three distinct

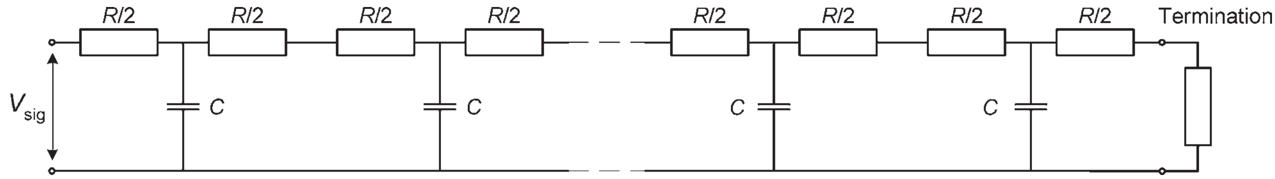

Fig. 2. Simplified schematic diagram of *RC* transmission-line ESL. Each T section of the transmission line, composed of a capacitor and two resistors, is highlighted for comparison with Fig. 6.

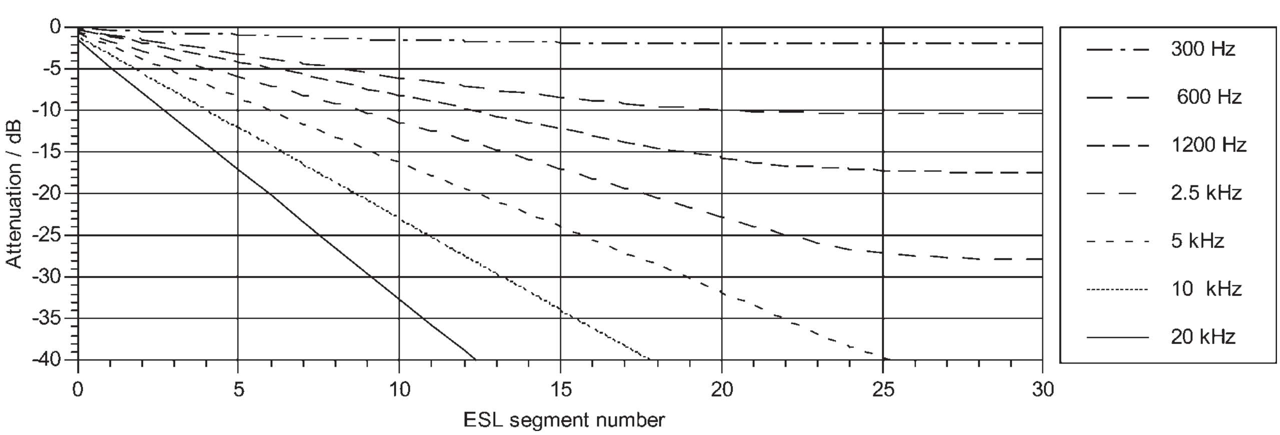

Fig. 3. Attenuation of stator voltages versus frequency for an unterminated 31-segment ESL with $R = 100$ kΩ and $C = 20$ pF. ESL is connected to amplifier, as in Fig. 2, at the zeroth (left-hand edge) segment.

operating regimes in the frequency domain. At very low frequencies the signal voltage propagates to the end of the transmission line practically unattenuated, and all stator segments move the diaphragm synchronously. In this regime the dominant impedance presented by an N-segment ESL is the total panel capacitance NC, so that the ESL current is

$$I_{\mathrm{sig}} = \mathrm{j}2\pi fNCV_{\mathrm{sig}} \tag{4}$$

and the SPL is found, using Eq. (1), to be

$$P_{\mathrm{LF}} = \frac{V_{\mathrm{sig}}V_{\mathrm{pol}}NC}{dh}\sqrt{\frac{f}{cr}}. \tag{5}$$

In this regime the SPL increases in proportion to the square root of the frequency (10 dB per decade).

In the midfrequency regime, by design, the ESL current is determined by the characteristic impedance of the transmission line [Eq. (3)], and the SPL is independent of frequency.

At the highest operating frequencies, because the impedance of the capacitors is very much less than that of the resistors, the ESL current flows only through the first stator segment and is determined by the first resistor in the transmission line ($R/2$ in Fig. 2),

$$I_{\mathrm{sig}} = \frac{2V_{\mathrm{sig}}}{R} \tag{6}$$

which leads to the SPL

$$P_{\mathrm{HF}} = \frac{V_{\mathrm{sig}}V_{\mathrm{pol}}}{R\pi dh\sqrt{crf}}. \tag{7}$$

The ESL current is independent of frequency so that the SPL falls as the square root of frequency (10 dB per decade).

Having calculated the SPL in each regime, the upper and lower cutoff frequencies of the ESL can now be determined. The transition between low- and mid-frequency regimes occurs when Eq. (3) and (5) give the same SPL,

$$f_{\mathrm{L}} = \frac{1}{2\pi N^2 RC}. \tag{8}$$

The frequency at which the mid- and high-frequency regimes give the same SPL is

$$f_{\mathrm{H}} = \frac{2}{\pi RC}. \tag{9}$$

The ratio of the two cutoff frequencies is

$$\frac{f_{\mathrm{H}}}{f_{\mathrm{L}}} = 4N^2 \tag{10}$$

which determines the minimum number of (equal-sized) ESL segments required to span a given bandwidth. For example, an ESL with a frequency response from 50 Hz to 20 kHz (a ratio of 1:400) must have a minimum of ten stator segments. A more nearly ideal ESL with a frequency response from, say, 20 Hz to 20 kHz requires 16 segments. Because N is the ratio of the total capacitance to the capacitance of the first segment, it also describes the relative-area requirement for an ESL using an RC transmission line with different sized segments. Fig. 4 plots an example of a calculation of the sensitivity of a 10-segment ESL versus frequency for a range of intersegment coupling resistances.

An important feature of Fig. 4 is the absence of SPL curves in the top left of the figure. That is, for a given total ESL capacitance NC, Eq. (5) with f replaced by f_{L} describes a fundamental compromise between the midband SPL and the lower cutoff frequency of the ESL.

In most ESLs a step-up transformer is used to match the voltage range of a solid-state amplifier to the higher voltage range required for the ESL. In that case V_{sig} in Eqs. (3), (5), and (7) should be replaced by ηV_{sig}, where η is the step-up ratio of the transformer. The step-up ratio is typically in the range from 50 to 200.

2 POLAR RESPONSE

The Walker equation [Eq. (1)] for the floor-to-ceiling doublet gives the on-axis SPL. As a listener moves horizontally off axis from a tall multisegment ESL, three different effects occur. First, the large-scale envelope of the SPL falls as cos(θ), where θ is the horizontal angle from the on-axis position. This is a consequence of the ESL being a dipole radiator [1], [3].

Second, on the smallest scale, radiation from different parts of a segment arrives out of phase, resulting in an attenuation factor given by the sinc function [10, p. 176]

$$\mathrm{sinc}\left(\frac{\pi fw}{c}\sin\theta\right) = \frac{\sin\left(\frac{\pi fw}{c}\sin\theta\right)}{\left(\frac{\pi fw}{c}\sin\theta\right)} \tag{11}$$

where w is the width of the segment. Fig. 5 plots Eq. (11) as a function of angle for segment widths of 30 and 10 mm, for different frequencies in the range of 2.5–20 kHz. Note that the 30-mm segment is narrower than the tweeter panels in many commercial multisegment ESLs, and shows the large variations in SPL (and frequency response) that occur with wide segments once the listener moves off axis. Note especially the pair of zeros near ±35° for the 20-kHz curve, indicating that a phase

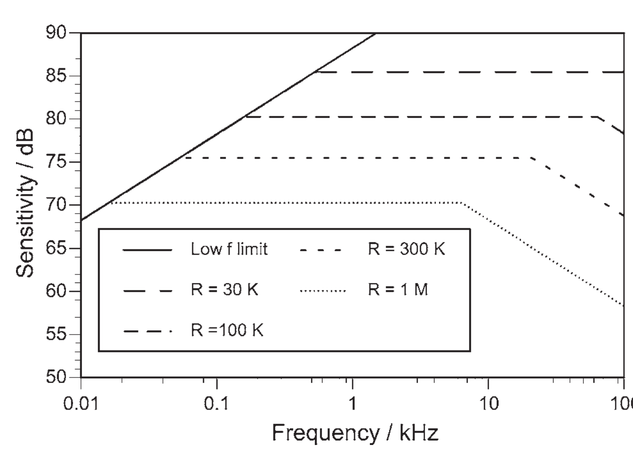

Fig. 4. Sensitivity of 10-segment ESL versus frequency and intersegment coupling resistance.

reversal occurs for angles beyond ±35°. A poor off-axis frequency response due to wide segments may be one factor contributing to the reputation of ESLs for having a narrow optimum listening position.

Eq. (11) shows that a zero-free polar response for all frequencies below 20 kHz requires the stator segments to be narrower than 17 mm. Fig. 5 suggests that a more nearly ideal polar response requires the stator segments to be no more than 10–12 mm wide. The maximum segment width, rather than bandwidth considerations [Eq. (10)], determines the number of segments required in a wide-range ESL. To achieve a satisfactory bass response (large ESL panel width) and a good high-frequency polar response, more than 30 equal-sized segments may be required.

The third contribution to the polar response arises from the sum of the various contributions to the SPL from each segment of a multisegment ESL. To calculate this sum, it is necessary to know the amplitude and relative phases of the currents in each segment. The voltage on the transmission line has the general form

$$V(n) = A\exp(-\gamma n) + B\exp(+\gamma n) \tag{12}$$

where n is the ESL segment number ($n = 0$ for the first segment), A and B are complex amplitudes determined by various operating parameters of the line, and γ is the attenuation constant,

$$\gamma = \sqrt{j2\pi fRC} = (1+j)\sqrt{\pi fRC}. \tag{13}$$

The mix of real and imaginary parts in Eq. (13) shows that the voltage experiences a combination of attenuation and phase rotation as it propagates along the transmission line. The first term of Eq. (12) describes the exponential decay in stator voltage as it propagates from the input of the transmission line. This term is evident in the linear parts of the curves on the left-hand side of Fig. 3. The second term of Eq. (12) describes the propagation of the signal reflected from the end of the transmission line, back toward the input. The combination of the two terms gives rise to the curvature and the flattening of the curves on the right-hand side of Fig. 3. Over most of the passband of the ESL the amplitude of the reflected signal is too small to contribute significantly to the SPL. This means that the stator voltage is well approximated by the first term of Eq. (12) with $V(0) = V_{sig}$, and the current through each stator segment is approximated by

$$I_{sig}(n) = V_{sig}\, j2\pi fC\exp(-\sqrt{j2\pi fRC}\,n). \tag{14}$$

The SPL at any listener position is obtained by summing the currents for all stator segments; each weighted according to the Walker equation [Eq. (1)], with an appropriate phase shift and attenuation arising from the transmission of the sound to the listening position [1], [2], [10]. In the far field (with respect to the width of the ESL) the horizontal polar response is approximated by neglecting the differences in amplitude due to distance and considering only the phase shifts [10]. With these approximations the far-field polar response is

$$\begin{aligned} P(\theta) &= \cos(\theta)\mathrm{sinc}\left(\frac{\pi fw}{c}\sin\theta\right)\frac{CV_{pol}V_{in}}{dh\sqrt{crf}} \\ &\quad\times\left|\sum_{n=0}^{N-1}\exp\left(-\sqrt{j2\pi fRC}\,n\right)\right. \\ &\quad\left.\times\exp\left(\frac{-j2\pi fnw}{c}\sin\theta\right)\right|. \end{aligned} \tag{15}$$

The summation in Eq. (15) is proportional to the Fourier transform of the current distribution in the ESL segments. Note also that the sign of the exponent in the second exponential term changes if the ESL transmission line runs right to left rather than left to right, as in Fig. 2 (that is, the sign of θ changes).

If the summation is extended from 0 to ∞ (which is a good approximation at most frequencies), and the sum is approximated by the corresponding integral, the far-field polar response is found to be

$$\begin{aligned} P(\theta) &= P(0)\cos(\theta)\mathrm{sinc}\left(\frac{\pi fw}{c}\sin\theta\right) \\ &\quad\times\left[\frac{2RCc^2}{RCc^2 + \left(\sqrt{RC}c \pm 2w\sqrt{\pi f}\sin\theta\right)^2}\right]^{1/2} \end{aligned} \tag{16}$$

where $P(0)$ is the on-axis response [Eq. (3)]. The sign in the denominator of the final term of Eq. (16) changes according to whether the ESL transmission line runs left to right or right to left. The asymmetric nature of the polar response is evident from the dependence of the denominator on the sign of sin θ, and hence the sign of θ.

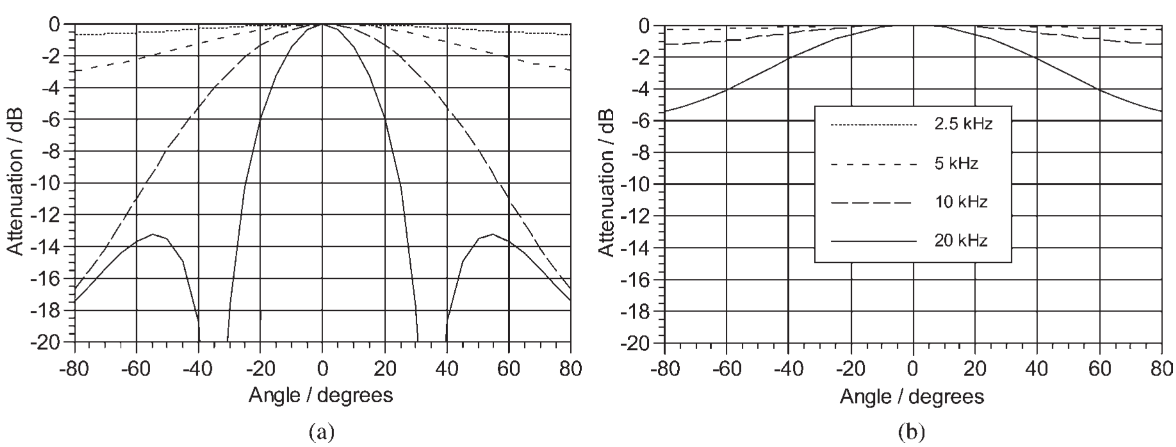

Fig. 5. Contribution of finite segment width to horizontal polar response of ESL. (a) 30-mm-wide strip. (b) 10-mm-wide strip.

A symmetric ESL can be constructed by driving an ESL at the center segment, rather than at one edge, and splitting the *RC* transmission line into two equal parts propagating left and right. Figs. 6 and 7 show the schematic diagram and the stator voltages for a symmetrical ESL. In order to use the same capacitance for each segment and maintain the same characteristic impedance (hence the same on-axis SPL and frequency response) as for the asymmetric case, the intersegment coupling resistance must increase by a factor of 4, and hence the attenuation constant [Eq. (13)] doubles. The far-field polar response of the symmetrical ESL is

$$P(\theta) = \cos(\theta)\mathrm{sinc}\left(\frac{\pi f w}{c}\sin\theta\right)\frac{CV_{\mathrm{pol}}V_{\mathrm{in}}}{dh\sqrt{crf}} \times \left|\sum_{(1-N)/2}^{(N-1)/2}\exp\left(-\sqrt{\mathrm{j}8\pi fRC}|n|\right) \times \exp\left(\frac{-\mathrm{j}n2\pi f w}{c}\sin\theta\right)\right|. \quad (17)$$

If the summation is approximated by the corresponding infinite integral, the polar response is found to be

$$P(\theta) = P(0)\cos(\theta)\mathrm{sinc}\left(\frac{\pi f w}{c}\sin\theta\right) \times \left(\frac{4R^2C^2c^4}{4R^2C^2c^4 + w^4\pi^2 f^2\sin^4\theta}\right)^{1/2}, \quad (18)$$

which is now independent of the sign of θ.

Fig. 8 shows the contribution of the ESL segment current distribution to the horizontal polar response, calculated using the final terms in Eqs. (16) and (18). Numerical calculations, made without approximation [except for Eq. (1)], show that Eqs. (16) and (18) describe the ESL polar response very well at typical listening distances.

The polar response of the asymmetric design is particularly disappointing with very rapid changes in SPL very close to the on-axis position ($\theta = 0$) and a 3-dB off-axis peak that moves with frequency. Many asymmetric multisegment ESLs using *RC* equalization probably have similar features in their polar responses, so this effect may be another factor contributing to the reputation of ESLs for a narrow optimum listening position. The observation may also explain why instructions for positioning ESLs sometimes recommend that the treble segment be placed on the inner edge of the ESL (for a stereo pair) and the loudspeakers be aligned slightly "toe in," so that the listener is positioned near the peaks, where the response is most uniform.

Of most interest is the symmetric ESL, which has a near perfect polar response at low frequencies and a remarkably broad response at the highest frequencies. The final term of Eq. (18) is a low-pass Butterworth filter response that is second order in sin θ and first order in frequency. Butterworth filters are maximally flat monotonic filters, and it is this that gives the symmetrical ESL a smooth zero-free polar response. The Butterworth response also

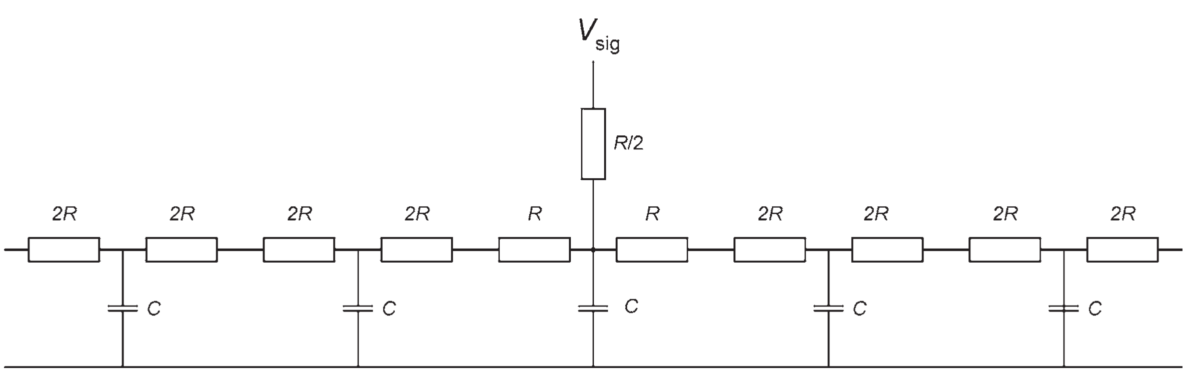

Fig. 6. Schematic diagram of symmetric *RC* transmission-line ESL with same characteristic impedance as in Fig. 1. Note different resistors values compared to those in Fig. 1.

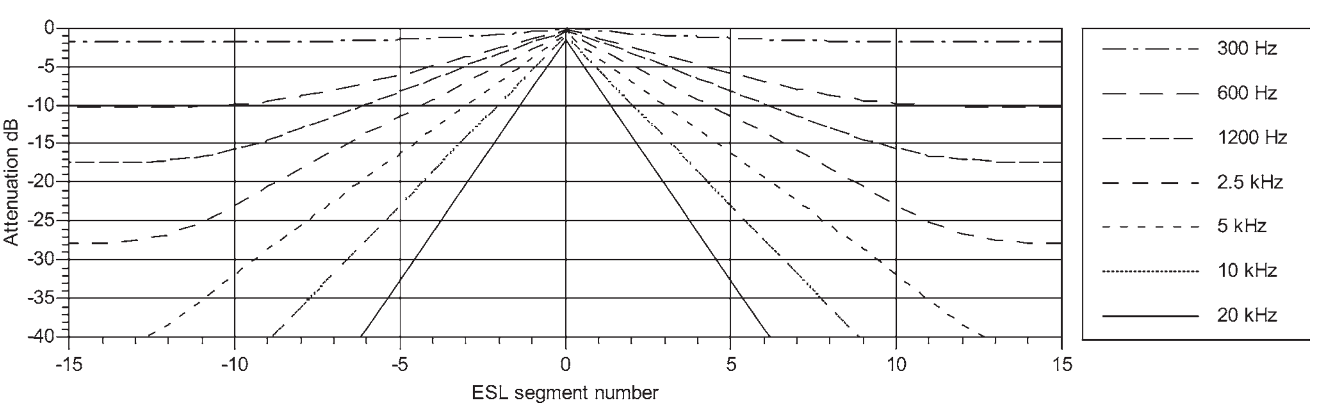

Fig. 7. Magnitude of stator voltage versus frequency for 31-segment ESL connected to amplifier, as in Fig. 6, at segment 0 (center segment). ESL is unterminated and has $R = 100\ \mathrm{k\Omega}$ and $C = 20$ pF.

ensures that the ESL is linear phase in the vicinity of the on-axis listening position.

Since the value of sin θ in Eq. (18) can be no greater than 1.0, for all frequencies below

$$f_0 = \frac{2RCc^2}{\pi w^2} \tag{19}$$

the contribution of the ESL segment current distribution to the polar response is practically flat to all angles. For an ESL with $R = 100$ kΩ, $C = 20$ pF, and $w = 12$ mm, f_0 is about 1 kHz.

Eq. (19) suggests that there is an advantage in using narrow segments. However, for a given ESL the capacitance per unit width of the segment C/w (per unit length of the transmission line) is more or less a fixed property determined by the diaphragm–stator spacing, and for a given SPL and frequency response, the intersegment coupling resistance is also a fixed property. That is, the polar response cannot be engineered independently of the frequency response, and there is a direct compromise between the angular extent of the high-frequency polar response, which broadens with increasing intersegment coupling resistance, and the SPL, which decreases with increasing resistance [Eq. (3)].

3 OTHER DESIGN CONSIDERATIONS

The preceding design equations give an idealized description of the performance of the ESL. These equations might suggest that achieving a flat extended frequency response is simply a matter of choosing the correct component values for the transmission line. In practice there are a large number of factors that must be considered, including the ESL panel dimensions and stator spacing, diaphragm tension and resonance, stator and diaphragm transparency, radiation impedance and diaphragm displacement, and the properties of the step-up transformer (if used). Most of these issues and other aspects of ESL engineering are discussed in [1], [3], [7], [8]. However, there are a few points relating to the implementation of the floor-to-ceiling doublet that warrant additional discussion.

3.1 Transmission-Line Termination

Section 1 gives the basic design equations for the ESL frequency response. However, these straight-line equations approximate a continuous curve and neglect the termination of the RC transmission line. A transmission line behaves differently when terminated with different impedances [9]. In all cases the voltage at any point on the line is given by Eqs. (12) and (13), but with values of the A and B parameters dependent on the termination. Fig. 9 shows an expanded plot of the low-frequency on-axis response of the ESL with a variety of different terminations on the transmission line.

The curves in Fig. 9 were calculated using a model of a 20-segment ESL that sums the currents in the different segments without approximation [other than Eq. (1)]. The solid line in Fig. 9 shows the frequency response inferred from Eqs. (3) and (5). First, if the transmission line is not terminated, the resulting reflection causes a slight (~1-dB) peak at low frequencies. With the transmission line terminated with the characteristic impedance to eliminate the reflection, the peak is reduced, but at the expense of the rolloff starting at a slightly higher frequency. The line indicated by the solid circles shows the frequency response when the line is terminated by an RC network simulating three additional sections of the transmission line.

Numerical experiments with ESLs with different numbers of segments indicated that a termination simulating 0.1–0.15N additional ESL segments ensures that the frequency response follows the straight-line equations closely.

3.2 Transformer Characteristics

Most ESLs employ a step-up transformer to match the voltage range of a solid-state power amplifier to the high voltages required for the ESL. The preceding straight-line equations not only approximate the ESL response at high

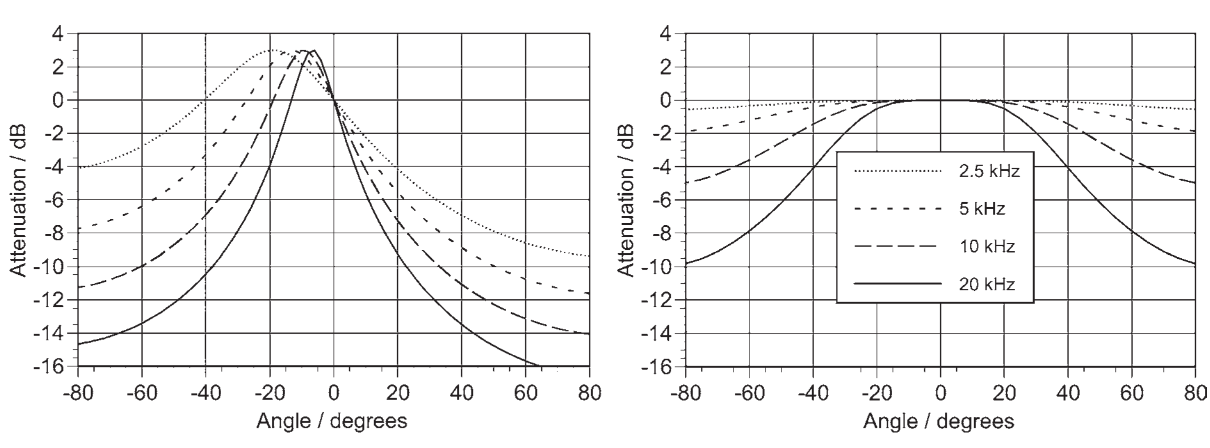

Fig. 8. Contribution of stator-current distribution to horizontal polar response. (a) Asymmetrical ESL. (b) Symmetrical ESL. Both ESLs have the same nominal sensitivity and frequency response.

frequencies, they neglect the resonance and second-order low-pass characteristics of the transformer.

One of the most undesirable characteristics of the transformer is the low input impedance presented to the amplifier when the series combination of the leakage inductance and the secondary-winding capacitance form a resonant short circuit. It seems likely that this is a major factor in the apparent difficulties with the use of solid-state amplifiers with ESLs. In addition in the absence of a suitable damping resistor, the resonance can be accompanied by significant frequency-response peaking, which is also undesirable.

The problems of amplifier load and response peaking can be addressed by both reflecting the leading $R/2$ resistor of the transmission line (Fig. 2 and 6) into the primary winding of the transformer, and connecting the transformer directly to the first stator segment. (The typical segment capacitance is no more than 30 pF, which will pull the resonant frequency of the transformer down a few percent.) The value of $R/2$ would normally be in the range 25–150 kΩ. Depending on the step-up ratio, the required primary resistance is typically a few ohms, but this becomes the limiting impedance only at frequencies beyond the transformer resonance. Ideally the resistance is adjusted (through either modeling or measurement) so that the combined response of transformer and ESL is as flat as practical.

In the midband of the ESL the impedance presented to the amplifier is the characteristic impedance of the *RC* transmission line [Eq. (2)] reflected into the primary circuit of the transformer Z_c/η^2, where η is the step-up ratio. The impedance is an equal mix of resistive and capacitive behavior (45° phase shift) and falls as the square root of frequency. Even with large transformer step-up ratios the magnitude of the impedance is typically greater than 4 Ω at 20 kHz.

3.3 Room Acoustics

With a symmetric (square or circular) ESL there is a clear distinction between near-field and far-field acoustic radiation regimes. In contrast, a rectangular ESL exhibits three different radiation regimes, as shown in Fig. 10.

The far field (regime I in Fig. 10) is the conventional operating regime for most loudspeakers. For an ESL the SPL is given by the original Walker equation [1], [2],

$$P = \frac{I_{\mathrm{sig}} V_{\mathrm{pol}}}{2\pi c r d}. \tag{20}$$

In this case the SPL decreases in proportion to the distance from the panel, and hence the acoustic energy falls as $1/r^2$, which is as expected for an expanding spherical wave.

The intermediate field (regime II in Fig. 10) is the operating regime for the floor-to-ceiling doublet. It is effectively in the far field with respect to the width of the ESL, and in the near field with respect to the height of the ESL. The SPL in this regime is given by Baxandall's alternative Walker equation [Eq. (1)]. The SPL decreases in proportion to the square root of the distance from the panel, and hence the acoustic energy falls as $1/r$, as expected for an expanding cylindrical wave.

The near-field (regime III in Fig. 10) is not normally used for loudspeakers. The SPL in this regime is given by another Walker equation,

$$P = \frac{I_{\mathrm{sig}} V_{\mathrm{pol}}}{2\pi d h w f} \tag{21}$$

where w is the width of the panel. In this case the SPL is independent of the distance from the panel, as expected for a plane wave.

The frequency where the loudspeaker behavior changes from one regime to another is determined by equating the expressions for SPL for the neighboring regimes. Hence the high-frequency transition occurs at

$$f_{\mathrm{w}} = \frac{cr}{w^2} \tag{22}$$

and the low-frequency transition at

$$f_{\mathrm{h}} = \frac{cr}{h^2}. \tag{23}$$

Note that both transition frequencies are dependent on the listener distance r. (Note also that Eqs. (22) and (23) are not based on the conventional criterion for distinguishing the near and far fields [5], [10].) Although not shown in

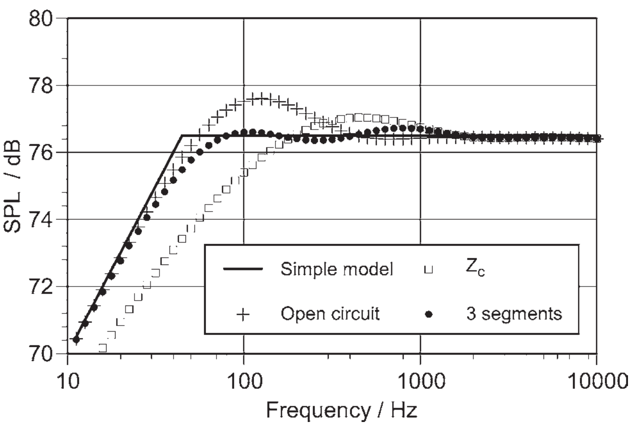

Fig. 9. Effect, on low-frequency response, of different terminations of the transmission line.

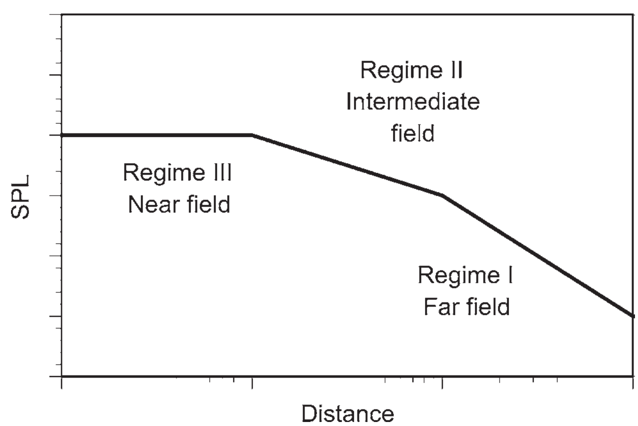

Fig. 10. SPL versus distance for rectangular ESL panel, showing three different radiation regimes.

Fig. 10, the transition from the intermediate field to the far field is also subject to ripples of about ± 2.5 dB due to diffraction and the abrupt end of the ESL panel [1], [5].

It is worthwhile considering a numerical example. For a segment 2.3 m high, 10 mm wide, and a listener distance of 3 m, the transition frequencies are $\sim$10 MHz and 193 Hz. While the higher transition frequency is not a design obstacle, the lower transition frequency seems to limit the ability of the loudspeaker to produce a useful low-frequency SPL.

A key assumption of the floor-to-ceiling design is that the response is augmented by reflections from the floor and the ceiling. In effect, the expanding cylindrical wave is trapped between floor and ceiling [1]. In practice, therefore, the limit to the low-frequency response of the ESL is determined not by the height of the ESL but by the acoustic reflectance of floor and ceiling. (The diaphragm resonance is another limit, but not discussed here.)

The reflectance of a membrane (thin relative to the wavelength) is described by the high-pass response [10]

$$|R(f,\alpha)|^2 = \frac{(\pi f \rho / z)^2 \cos^2 \alpha}{1 + (\pi f \rho / z)^2 \cos^2 \alpha} \tag{24}$$

where α is the angle of incidence of the sound wave on the reflecting surface, ρ is the density of the membrane in kg/m^2, and z is the acoustic impedance of air. For a wall of density 10 kg/m^2, which is typical of timber frame and flooring panel construction, the cutoff frequency is about 15 Hz. For example, the apparent height of the 2.3-m ESL with a single reflection from each of the floor and the ceiling is 6.9 m, and the cylindrical wave regime then extends to 21 Hz. Additional reflections extend the response rapidly to the limit imposed by the reflectance [Eq. (24)]. Thus buildings with a lightweight timber construction will generally have sufficient mass to extend the bass response to the lowest practical frequencies and eliminate any diffraction effects.

3.4 Maximum SPL

One of the key performance parameters of an ESL is the maximum SPL. Baxandall [1, pp. 156–157] derives an expression relating the maximum SPL, $P_{\max}$, to area and frequency for the conventional ESL operating in the far field,

$$P_{\max} = \left(\frac{F}{A}\right)_{\max} \frac{A}{2} \frac{f}{cr} \tag{25}$$

where A is the area of the stator, and $(F/A)_{\max}$ is the maximum force per unit area that can be applied to the diaphragm before the air dielectric breaks down. Baxandall determined that $(F/A)_{\max}$ is about 50 N/m^2. Because $P_{\max}$ increases with frequency, the practical limitations in SPL occur at the lowest operating frequencies. For a stator of 0.5-m^2 area at 100 Hz and 1 m listening distance, the maximum SPL is 105.3 dB. Note that because of perforations the area of the stator is normally about 50% of the panel area.

The same analysis applied to the floor-to-ceiling doublet yields the analogous relation

$$P_{\max} = \left(\frac{F}{A}\right)_{\max} \frac{A}{2h} \sqrt{\frac{f}{cr}} \tag{26}$$

which shows a weaker dependence on frequency and listening distance, and a dependence on the width ($=A/h$) of the ESL panel rather than its area. For a stator 2.3 m high and 0.5 m^2 in area, the maximum SPL is 104.6 dB, a little lower than the conventional ESL. However, such a comparison is misleading because the conventional ESL is modeled as a point source whereas the floor-to-ceiling doublet is a line source: at practical listening distances, the line source will yield the larger SPL. At 3-m listening distance the maxima are 95.7 dB and 98.6 dB respectively, with the line source giving the greater SPL.

A comparison of Eqs. (25) and (26) shows that for frequencies below cr/h^2 the floor-to-ceiling doublet has a larger maximum SPL for a given stator area. This is the same frequency [Eq. (23)] at which the reflections from floor and ceiling begin to reinforce the SPL.

3.5 Practical Realization

In appearance an ESL based on the floor-to-ceiling doublet would be similar to the tallest of the conventional multisegment ESLs: typically a 50–70-mm thick panel, 200–600 mm wide, extending from the floor to the ceiling. It would also have a very similar frequency response and maximum SPL. Only the polar response would be significantly different.

The large number and narrow width of segments makes the stators of the floor-to-ceiling doublet more complicated than those in many conventional ESLs and reduces the options for construction. However, there are two established construction techniques well suited to the floor-to-ceiling doublet. The first technique, employed by Quad in the ESL-63, utilizes large printed circuit boards with the stator pattern etched in the copper [1]. The PCB is also convenient for mounting the intersegment coupling resistors. Second, the wire–stator structure used by Malme [6] and recommended by Sanders [7] is readily adapted to the floor-to-ceiling doublet. In either case the drive signal to each stator is connected to the middle segment to give the ESL the symmetrical zero-free polar response. If required, the two segments at the edges of each stator are connected to the terminating network to fine-tune the low-frequency response (Section 3.1).

Other possible construction methods include using an insulating substrate for the stator, like the PCB, but with foil tapes or conductive paint placed in the appropriate pattern. One interesting possibility is to use paint with a high and uniform resistivity in the range from about 10–100 MΩ per square to cover the entire stator as a single rectangular block. The layer would simultaneously form both the capacitive and the resistive parts of the transmission line. This would eliminate the finite segment width [Eq. (11)] from the polar response and maximize the stator area. Connections could be made via metal tape or wire embedded under the paint.

4 DISCUSSION AND CONCLUSIONS

A symmetrical *RC* transmission line is used to equalize the frequency response of a multisegment floor-to-ceiling electrostatic loudspeaker. Section 1 gives the design equations for the SPL and frequency response, including the upper and lower cutoff frequencies, and Section 2 describes the three contributions to the horizontal polar response. The scheme provides accurate equalization over a very wide range of frequencies, listening distances, and angles.

Because most of the design parameters are constrained to a narrow range of values by engineering concerns, the two most important design parameters are the total capacitance (size) of the ESL and the intersegment coupling resistance of the transmission line. The total capacitance determines the limit of the compromise between the midband sensitivity and the bass cutoff frequency of the ESL, as described by Eq. (5). The intersegment coupling resistance then determines the midband sensitivity and the high-frequency polar response of the ESL. Eq. (5), because it describes the minimum panel capacitance required to achieve a given SPL and bass response, is possibly the most fundamental design equation. A similar equation, derived from the first Walker equation, applies to conventional ESLs operating in the far field:

$$P = \frac{V_{\mathrm{sig}} V_{\mathrm{pol}} f_{\mathrm{L}} C_{\mathrm{total}}}{crd}. \tag{27}$$

This equation does not appear to have been reported previously.

Three terms make up the polar response of the ESL: a cosine term due to the dipole nature of the ESL, a sinc term due to the finite width of the stator segments, and a second-order Butterworth term due to the distribution of the stator current in the different segments. The loudspeaker, with perhaps 20 to 40 segments, is similar to the parametric line arrays used for beamforming [10]. The symmetrical version of the *RC* transmission line provides a stator-current distribution in the array of ESL segments of the exact form required to generate the Butterworth polar response. Kinsler et al. [10, p. 490] also give an example of an ultrasonic transducer for marine applications with the same second-order Butterworth polar response.

ESLs tend to have a poorer bass response than most other loudspeaker technologies. The floor-to-ceiling doublet has several minor advantages in this respect. First, the *RC* transmission-line equalization ensures that the entire ESL panel contributes to the acoustic output at low frequencies, and hence achieves the maximum low-frequency SPL for a given panel area. Second, the floor-to-ceiling design has a smaller footprint for a given panel area than other ESL designs. Third, because the ESL is a line source, the SPL falls as the square root of distance rather than distance, as for most other loudspeakers. This means that the SPL at practical listening distances can be more than 6 dB higher than sensitivity figures would imply. Finally, the reflections from the floor and ceiling are used to augment the response at low frequencies. This may also be true for other wide-range ESLs, although probably based more on empirical observations than theoretical principle, as is the case here.

ESLs also have a reputation for causing solid-state amplifiers to operate outside the safe operating area of the output transistors. The problem is usually attributed to a combination of the low impedance and the predominantly capacitive load of the ESL. In fact, part of the problem may be the short circuit formed by the transformer leakage inductance and winding capacitance at resonance. The *RC* transmission line mitigates these effects in two ways. First, the ESL impedance is proportional to the characteristic impedance of the transmission line, which falls as the square root of frequency to relatively high values compared to conventional purely capacitive ESLs. Second, the leading resistor of the transmission line can be reflected into the primary circuit of the transformer to isolate the amplifier from the transformer.

A loudspeaker of the proposed design has yet to be built and tested. However, as much as practical, the various design equations have been tested thoroughly using computer models. Indeed, the theoretical analysis leading to Eqs. (16) and (18) was prompted by the unexpected (poor and good) computed responses of asymmetrical and symmetrical ESLs.

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