Spotlight

AES "Classics" Series

Editor's Note: The Audio Engineering Society has graciously given Voice Coil *permission to reprint previous convention papers (preprints) as well as* Journal of the Audio Engineering Society *(JAES) articles. My intention is to feature papers I consider particularly interesting and valuable to loudspeaker engineers, the past "classics" published during the long and rich history of AES Convention paper presentations. I hope these will be a welcome addition to* Voice Coil.

For the first article, I chose a paper written by loudspeaker engineering icon Neville Thiele titled, "Closed Box Loudspeaker with a Series Capacitor," originally published in the JAES *(Volume 58, Issue 7/8, pp. 577-582, July 2010). The simple but effective technique enables you to achieve a lower F3 in a sealed box for a given volume with the addition of a large value capacitor (typically 250-to-400-µF NPs) and is useful with small 3-to-5.25" woofers. I have used this technique in production loudspeakers, and I saw it applied in several well-known manufacturers' products.*

Again, I want to thank the Audio Engineering Society, JAES *editor Bozena Kostek, and* JAES *managing editor Bill McQuade for allowing* Voice Coil *permission to do this.*

Sadly, as this issue was "going to press," I was informed by the AES that Neville Thiele had passed on October 1, 2012. While I will feature a tribute to Mr. Thiele and his amazing work in Voice Coil*'s December issue, I can think of no more fitting tribute than to republish one of Neville's* JAES *articles on the month of his passing (*Voice Coil*'s November issue was written late September, early October).*

I am not including the references with this series. A complete PDF of this paper, including references, can be obtained from the AES website's e-library (www.aes.org).

Closed-Box Loudspeaker with a Series Capacitor*

By Neville Thiele, AES Fellow
Faculty of Architecture, Design and Planning,
University of Sydney, Sydney, NSW, Australia

The connection of a capacitor in series with a closed-box loudspeaker extends its response at lower frequencies with a smaller box. At the same time it confers worthwhile protection against excessive excursion of its voice coil from subsonic input signals, which produce no useful acoustic output. A design procedure and suitable transfer functions are presented.

INTRODUCTION

Once the parameters of a loudspeaker driver are known, its electrical-to-acoustical transfer function can be determined in the same manner as a conventional electrical high-pass filter. When the rear of the driver is enclosed in a sealed box, the transfer function is of second order. Insertion of appropriate reactive elements between the amplifier and the driver can change this transfer function to one of higher order. For the simplest case of a series capacitor, the second order function changes to third order. In that case, at signal frequencies below the resonance of the driver, its impedance, having peaked and presented a resistance, falls and includes a component of positive, that is, inductive, reactance. A series capacitance presents a negative reactance, which tends to cancel the driver reactance, thus increasing in this region the electric power that the driver absorbs from the input and radiates in its acoustical output.

This addition of electrical high-pass element(s) confers two advantages. It protects the driver from input signals at frequencies below its cutoff, which otherwise would produce excessive voice-coil excursions while producing little or no acoustic output. At the same time it extends the useful low frequency range to a small but significant extent.

While this procedure enhances the performance of a closed box, it cannot help a vented box, whose bottom response limit occurs around its box resonance. At that frequency the driver impedance goes through a resistive minimum. At lower frequencies it rises, presenting a capacitive reactance component that gains no advantage from a series capacitance.

The series capacitor may be considered a kind of equalizer, but from the author's point of view it is an integral part of the system design, along with the parameters of the driver and box. This use of series capacitance was described first by Benson [1] and later by von Recklinghausen [2] and Woodgate [3], but to the author's knowledge, no closed form procedure for designing such a system has been offered. This engineering report hopes to fill that gap.

Earlier publications [4,5] have considered systems where a closed-back driver, whose electrical-to-acoustic transfer function is that of a second order high-pass filter, is embedded with second and third order electrical filters to produce overall transfer functions of fourth and fifth orders, respectively. Those papers describe designs primarily in the context of crossover systems, whose transition frequencies are most often higher than 1,000 Hz, where the electrical filter components are comparatively small in size and cost. Nevertheless the principle applies equally well, as it does in this study, to lower transition frequencies if the size and cost are justified.

DISCUSSION

A loudspeaker comprising a driver mounted on an infinite baffle has a second order high-pass transfer function, which relates its acoustic output within the piston range to its electrical input voltage:

$$F(sT_S) = \frac{s^2T_S^{\,2}}{s^2T_S^{\,2} + sT_S\left(\dfrac{1}{Q_{MS}} + \dfrac{1}{Q_{ES}}\right) + 1} \qquad (1)$$

where T_S is the characteristic time constant of the driver ($= 1/2\pi f_S$, with f_S being its resonance frequency), and Q_{MS} amd Q_{ES} are its mechanical and electrical quality factors, respectively.

It can be shown that the response of a loudspeaker can be modeled by the current flowing in the capacitance branch of the motional impedance of its electrical equivalent circuit, as shown in **Figure 1**, which is derived and simplified from the mechanical equivalent circuit of Figure 8.4 [6] (see [7]). When this driver, whose equivalent air volume is V_{AS}, is placed in an enclosed box of volume V_B its transfer function is modified to:

$$F\left(sT_S\right) = \frac{s^2T_S^{\ 2}}{s^2T_S^{\ 2} + sT_S\left(\frac{1}{Q_{MS}} + \frac{1}{Q_{ES}}\right) + \left(1 + \frac{V_{AS}}{V_B}\right)} \quad (2)$$

This is more easily handled as a system by dividing both its numerator and its denominator by $(1 + V_{AS}/V_B)$ and nominating new parameters, now of the resulting driver plus closed box system:

$$f_{SC} = f_S\sqrt{1 + \frac{V_{AS}}{V_B}} \quad (3)$$

$$T_{SC} = \frac{T_S}{\sqrt{1 + \frac{V_{AS}}{V_B}}} \quad (4)$$

$$Q_{MC} = Q_{MS}\sqrt{1 + \frac{V_{AS}}{V_B}} \quad (5)$$

$$Q_{EC} = Q_{ES}\sqrt{1 + \frac{V_{AS}}{V_B}} \quad (6)$$

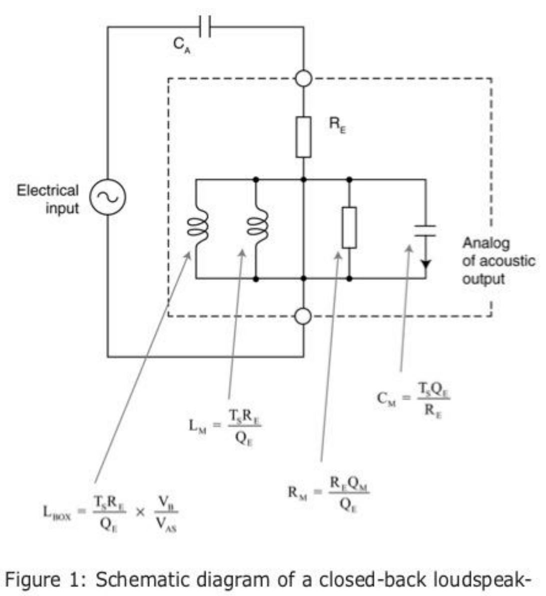

Figure 1: Schematic diagram of a closed-back loudspeaker fed through series capacitance C_A. Dashed box encloses a model of the driver plus box. L_M, R_M, and C_M—components of the driver motional impedance; L_{BOX}—electrical impedance contributed by V_B box volume.

and applying these parameters to a new second order highpass function for the system:

$$F\left(sT_{SC}\right) = \frac{s^2T_{SC}^{\ 2}}{s^2T_{SC}^{\ 2} + sT_{SC}\left(\frac{1}{Q_{MC}} + \frac{1}{Q_{EC}}\right) + 1} \quad (7)$$

which is the same as Eq. (1), except that its characteristic frequency f_{SC} and its two Qs are increased $\sqrt{(1 + V_{AS}/V_B)}$ times. If now this loudspeaker system is mounted in a box of volume V_B and fed through a capacitance C_A, its transfer function becomes:

$$F\left(sT_{SCC}\right) = \frac{s^3T_{SC}^{\ 2}T_A}{sT_{SC}^{\ 2} + s^2\left[T_{SC}^{\ 2} + T_AT_{SC}\left(\frac{1}{Q_{EC}} + \frac{1}{Q_{MC}}\right)\right] + s\left(\frac{T_{SC}}{Q_{MC}} + T_A\right) + 1} \quad (8)$$

where T_A is the product C_AR_E. [We could have written the sum $1/Q_{EC} + 1/Q_{MC}$ as $1/Q_{TC}$, but prefer to keep the two Qs separate until Eq. (10).]

We now rewrite Eq. (8) in terms of the primary parameters of the driver and box by nominating:

$$k^2 = \frac{1}{1 + \frac{V_{AS}}{V_B}} \quad (9)$$

and writing $k^2T_S^2$ [$= k^2/(2\pi f_S^2)$] for T_{SC}^2, Q_{MS}/k for Q_{MC}, and Q_{ES}/k for Q_{EC}. Then Eq. (8) becomes:

$$F\left(sT_{SCC}\right) = \frac{s^3k^2T_S^{\ 2}T_A}{s^3k^2T_S^{\ 2}T_A + s^2\left(k^2T_S^{\ 2} + \frac{k^2T_AT_S}{Q_{TS}}\right) + s\left(\frac{k^2T_S}{Q_{MS}} + T_A\right) + 1} \quad (10)$$

To constrain this response to a desired response whose transfer function is:

$$F(sT) = \frac{x_3r^3s^3T_S^{\ 3}}{x_3r^3s^3T_S^{\ 3} + x_2r^2s^2T_S^{\ 2} + x_1rsT_S + 1} \quad (11)$$

where:

$$r^3T_S^{\ 3} = k^2T_S^{\ 2}T_A \quad (12)$$

We write:

$$t = \frac{T_A}{T_S} \quad (13)$$

and we match coefficients for the various powers of sT_S. The coefficients x_1, x_2, and x_3 are chosen by the designer to produce the desired shape of the response. If x_0, the coefficient of the zeroth-order term, is not 1 as it is in Eq. (11), all the coefficients x_0, x_1, x_2, and x_3 in the numerator and denominator of the function in Eq. (11) can be divided by x_0 without changing its response.

Then:

$$k^2t = x_3r^3 \quad (14)$$

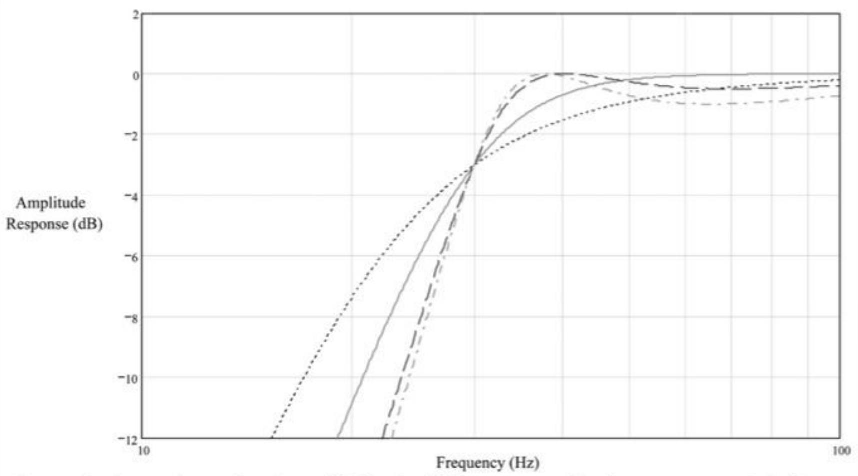

Figure 2: Comparison of various third-order high-pass amplitude responses. −3.0 dB at 30 Hz. — Butterworth; ——— Chebyshev with 0.5-dB ripple; –•– Chebyshev with 1.0 dB ripple; ••• Bessel.

$$k^2\left(\frac{t}{Q_{TS}}+1\right)=x_2r^2 \tag{15}$$

$$t+\frac{k_2}{Q_{MS}}=x_1r \tag{16}$$

and then:

$$k^2=\frac{x_3r^3}{t} \tag{17}$$

$$t=\frac{1}{\dfrac{x_2}{x_3r}-\dfrac{1}{Q_{TS}}} \tag{18}$$

and:

$$x^2_3r^3 - 2x_2x_3Q_{TS}r^2 + rQ_{TS}\left(x^2_2Q_{TS} + x_1x_3Q_{MS}\right) + Q_{MS}Q^2_{TS}\left(x_3 - x_1x_2\right) = 0 \tag{19}$$

The cubic Eq. (19) is solved for r. Of the methods available, the author prefers the iterative Newton–Raphson method [8] starting with an initial guess of 1 for r. In all cases investigated so far, only one positive real root was found, so no further calculation of r was necessary. The value of r, the ratio in which the resonance frequency f_S and its characteristic time constant T_S are changed to the new characteristic time constant rT_S, is substituted in Eq. (18) to find t, hence also T_A from Eq. (13). These values of r and t are substituted in Eq. (17) to find k^2, hence the ratio V_{AS}/V_B from Eq. (9), and then the required box volume V_B. Finally the value of the series capacitance C_A is:

$$C_A=\frac{T_A}{R_E} \tag{20}$$

USEFUL RESPONSE FUNCTIONS

The response shapes of a loudspeaker system most usually desired bear the names of Butterworth, Chebyshev, and Bessel. Their third order low-pass versions are most familiar in the forms of Eqs. (21), (22), and (23), though later we will be using the high-pass versions.

- Third-order Butterworth, low-pass:

$$F(s)=\frac{1}{1+2sT_0+2s^2T^2_0+s^3T^3_0} \tag{21}$$

- Third-order Chebyshev, low-pass:

$$F(s)=\frac{\left(1+\dfrac{3K}{4}\right)}{\left(1+\dfrac{3K}{4}\right)+\left(2+\dfrac{3K}{4}\right)sT_0+2s^2T^2_0+s^3T^3_0} \tag{22}$$

- Third-order Bessel, low-pass:

$$F(s)=\frac{15}{15+15sT_0+6s^2T^2_0+s^3T^3_0} \tag{23}$$

The most generally useful function is the Butterworth, which produces a maximally flat amplitude response. Of all possible functions, its response inband remains the longest near reference level. At its characteristic frequency, when $\omega T_0 = 1$, that is $f = f_0$, its response is 3.0 dB below reference level. At frequencies out of band its response remains closest to the asymptotic slope of all third-order functions, namely, 18 dB per octave (60 dB per decade).

The equal ripple Chebyshev response may be defined algebraically [9] as in Eq. (22), where the constant K determines the amount of ripple. The squared magnitude

Transfer Function Type	$f_{-3dB} = f_0$ (Hz)	V_B (liters)	Q_{TC}	T_A (µs)	C_A (µF)
Butterworth	67.7	11.2	0.936	4502	900
Chebyshev (0.5-dB ripple)	66.6	9.7	0.992	2385	477
Chebyshev (1.0-dB ripple)	64.6	10.5	0.959	2031	406
Bessel	68.6	18.8	0.766	7593	1519
*driver parameters are f_S = 40 Hz; Q_{MS} = 3.6; Q_{ES} = 0.45; Q_{TS} = 0.40; V_{AS} = 50 liters; R_E = 5Ω					

Table 1: Parameters required for various transfer functions

$|F(j\omega)^2|$ of its low-pass response ripples, as in the dashed and dash–dot curves of **Figure 2**, between a maximum of 1 (0 dB) at high frequencies inband and also when $\omega^2T_0^2 = 3/4K$ and a minimum of $1/[1 + K^3/(4 + 3K)^2]$ when $\omega^2T_0^2 = ¼K$ and also K. When $\omega^2T_0^2 = K + 1$ the squared magnitude goes through a value of $1/[2 + K^3/(4 + 3K)^2]$ that is not greatly different from −3.0 dB, that is, between −3.0 and −3.7 dB, at commonly used values of K between 0 and 5. When K diminishes to zero, the Chebyshev function degenerates to a Butterworth.

However, for the present application it has proved more useful to take two values of K, 2.5481 and 4.0949, which produce ripple amplitudes of 0.5 and 1.0 dB, respectively, and then adjust the resulting coefficients in Eqs. (25) and (26) according to their powers of sT_0 so that the magnitude of both responses is −3.0 dB when $\omega T_0 = 1$.

• Butterworth third-order, high-pass:

$$F(s) = \frac{s^3T^3_0}{1 + 2sT_0 + 2s^2T^2_0 + s^3T^3_0} \qquad (24)$$

• Chebyshev third-order, high-pass (0.5-dB ripple; −3.0 dB when $\omega T_0 = 1$):

$$F(s) = \frac{0.4498s^3T^3_0}{1 + 1.7032sT_0 + 1.1261s^2T^2_0 + 0.4498s^3T^3_0} \qquad (25)$$

• Chebyshev third-order, high-pass (1.0-db ripple; −3.0 dB when $\omega T_0 = 1$):

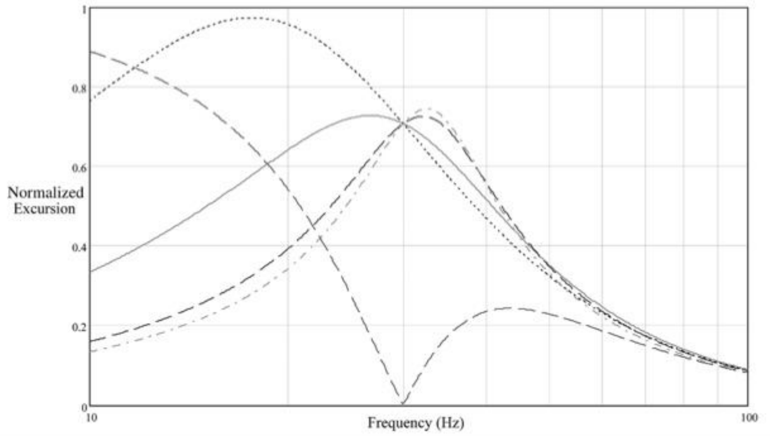

Figure 3: Comparison of driver excursions, various third-order high-pass responses. −3.0 dB at 30 Hz. — Butterworth; ———Chebyshev with 0.5-dB ripple; -•— Chebyshev with 1.0-dB ripple; ••• Bessel; ——— fourth-order high-pass Butterworth, from vented box (for comparison).

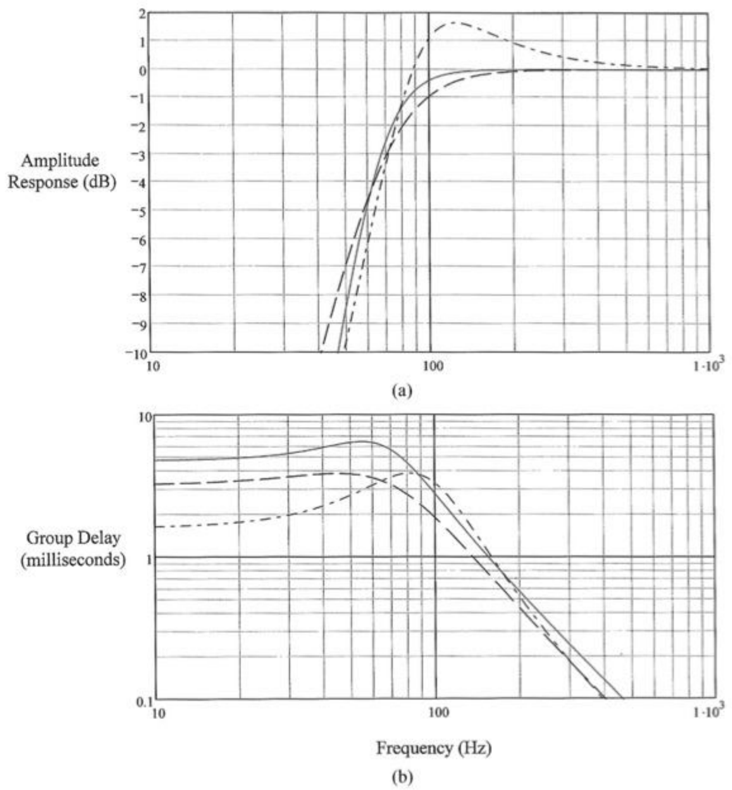

Figure 4: Responses of driver of Table 1, resonance F_S = 40 Hz; Q_T = 0.40; V_{AS} = 50 liters. (a) Amplitude versus frequency. (b) Group delay versus frequency. —in 11.2-liter box and with a 900-µF capacitor in series, third-order Butterworth response; -•- in 11.2-liter box without series capacitor; produces second-order response with 1.6-dB peak; ——— in 23.5-liter box without series capacitor, for second-order Butterworth response.

$$F(s) = \frac{0.3743s^3T_0^3}{1 + 0.9027sT_0 + 1.0331s^2T_0^2 + 0.3743s^3T_0^3} \qquad (26)$$

The coefficients of the Bessel (maximally flat delay) response were adjusted likewise from the low-pass function of Eq. (23) to the high-pass function Eq. (27), whose response has a magnitude of–3.0 dB when $\omega T_0 = 1$.

• Bessel third-order, high-pass (–3.0 dB when $\omega T_0 = 1$):

$$F(s) = \frac{2.6237s^3T_0^3}{1 + 3.4015sT_0 + 4.6280s^2T_0^2 + 2.6237s^3T_0^3} \qquad (27)$$

It should be remembered that only one set ("alignment") of parameters can produce a given transfer function from a given driver. The values of the parameters will vary with a number of factors—the closeness of the driver's initial Q_T to the Q_T that the function requires, and to some extent, the ratio of Q_M to Q_E.

The alignments in **Table 1** give some insight into the parameter values required with a typical driver to produce each of the four preceding responses above. Q_{TC} is the Q_T of the driver + box system. The capacitance values needed for C_A are comparatively large, in the hundreds of microfarads. The capacitor needs to be chosen for a low series loss resistance, small compared with the resistance of R_E, since its contribution to the total resistance in the circuit effectively increases Q_E in the same proportion.

The responses of the four functions are plotted in **Figure 2** for responses that are all –3.0 dB at 30 Hz (for which T_0 = 5305 μs). They demonstrate the much faster cutoff of the Chebyshev responses in the stop-band, though the improvement in going from 0.5-to 1.0-dB ripple is comparatively small. They also show how the Bessel's amplitude response droops slowly across the pass-band.

The voice-coil excursions for these responses are plotted in **Figure 3**, and compared with the excursion of a ventedbox loudspeaker with a fourth-order Butterworth response. All curves are normalized to a maximum value of 1 for the excursion of the driver at very low frequencies with a standard level of input signal in the passband.

It should be remembered, though, that the driver's voice coil excursion at very low frequencies has already been reduced $1/[1 + (V_{AS}/V_B)]$ times below its unbaffled value, the system compliance being stiffened by the air enclosed in the box.

Table 1 shows how the use of a series capacitor with a suitable driver allows good bass response to be launched from a surprisingly small box. **Figure 4** shows how great the saving in box volume can be, 11.2 versus 23.5 liters, to less than half in this example.

The group delay plot of **Figure 4** should be read in light of [10], where the threshold of audibility of the group delay is reported as increasing from a minimum of 1 ms at 2 kHz to 2.0 ms at 8 kHz and 3.2 ms at 500 Hz, with no figures for thresholds beyond those frequencies. At the same time **Figure 3** shows how much protection the additional highpass filtering confers against excessive excursion at very low frequencies. While a vented box has the clear advantage over all the closed-box responses that its cone excursion is much less in the region of cutoff, the designer must weigh this against the problems of increasing size and cost in providing a vent for a small box at low frequencies, or of a passive radiator.

CONCLUSION

When a capacitor is inserted in series with a closed-box loudspeaker, it extends the response to lower frequencies and enables the use of a much smaller box. At the same time it confers substantial protection against excessive excursion of the voice coil from subsonic input signals that would produce no useful acoustic output. The design procedure is straightforward and is easily adapted to desirable transfer functions.

ACKNOWLEDGMENT

The author gratefully acknowledges Graeme Huon's experimental verification of the design procedure and his helpful comments, along with those of the Journal's reviewers, regarding the text.

* The manuscript was received April 28, 2010; revised May 25, 2010 to run in the *JAES*.

THE AUTHOR

The late A. Neville Thiele graduated from the University of Sydney in 1952 with a Bachelor of Engineering (Mechanical and Electrical) degree.

From 1952 to 1961, at E.M.I. Ltd. (Australia), he worked on the development of telemetry, radio and television receivers, and electronic test equipment. In 1962, he joined the Australian Broadcasting Corp., where he designed and assessed equipment and systems for sound and television broadcasting. From 1980 until his retirement in 1985, Mr. Thiele was its director of engineering research and development. Until his death, he was a consulting engineer in the fields of audio, radio, and television, and he taught in the University of Sydney graduate audio program.

Mr. Thiele published more 40 papers about loudspeakers, filters, and equalizers, and about testing methods for sound and video broadcasting. He helped set national and international standards, serving on Audio Engineering Society (AES) committees, ITU-R, and through Standards Australia. He was a Fellow of the AES and the Institution of Engineers, Australia, and a member of the Society of Motion Picture and Television Engineers. He was president of the Institution of Radio and Electronics Engineers Australia from 1986 to 1988 and vice president, AES's international region from 1991 to 1993 and from 2001 to 2005. **VC**